

A. Properties of Exponents

$a^m \cdot a^n = a^{m+n}$ if you multiply the bases, add the exponents

$(a^m)^n = a^{mn}$ if you raise a power to another exponent, multiply the exponents

$\frac{a^m}{a^n} = a^{m-n}$ if you divide the bases, subtract the exponents

$a^0 = 1$ $a^{-n} = \frac{1}{a^n}$ a negative exponent is the symbol to take the reciprocal

1. $\frac{18w^5x^3}{6w^2x}$ 2. $\frac{20ab^9c^2}{12a^3b^2c^2}$ 3. $(5x^4y^3z^8)^0$ 4. $5x^3(2x^7)$

5. $(4x^{-3}y^5z)^3$ 6. $(2x^4y^3)^2(3xy^5)^3$ 7. $(5x^3)^2(2x^5)^2$

8. examine the solution #7. Identify the following:

Coefficient _____ base _____ exponent _____

Write each of the following with a single power:

9. $100x^2y^2$ 10. $27x^4y^3$ 11. $125a^3b^9$ 12. $36x^4y^2$

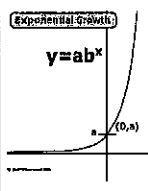
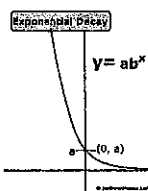
Solve for x: (hint - identify the base that is larger and rewrite it as a power.)

13. $2^{4x} = 32^2$ 14. $3^{8x} = 81^{12}$ 15. $2^x = 64^6$

20. Eppler's centipede population is doubling every 5 months. Right now there are 3 centipedes. How many will there be in 60 months?

21. The half life of Byron-19 is 3 years. If you have 100 mg of Byron-19 right now:

- A. what is an equation that models this situation? _____
- B. How much Byron-19 will there be 12 years? _____
- C. How much Byron-19 will there be in 36 years? _____
- D. How much Byron-19 was there 6 years ago? _____

| | |
|---|--|
| <p>Exponential growth</p> <p>$Y = ab^x$</p> <p>a = initial/starting amount</p> <p>b = growth factor, $b > 1$</p> <p><small>Exponential Growth</small></p> <p>$y = ab^x$</p>  | <p>Exponential decay</p> <p>$Y = ab^x$</p> <p>a = initial/starting amount</p> <p>b = decay factor, $0 < b < 1$</p> <p><small>Exponential Decay</small></p> <p>$y = ab^x$</p>  |
|---|--|

16. Evaluate $y = 4^x$ for the domain $\{-2, -1, 0, 1, 2, 3\}$ No decimals!

17. Compound interest: you can use $y = ab^x$ OR $B = P(1 + r)^x$

a and P = starting amount

b or r = interest rate per period $\frac{\%}{\text{number of times per year compounded}}$

x = number of times interest is compounded $(\text{number of years passed} \cdot \text{number of times per year compounded})$

A. Invest \$500 at 3% annual for 12 years

B. invest \$500 at 3% semiannual for 12 years

C. invest \$500 at 3% quarterly for 12 years

D. invest \$500 at 3% monthly for 12 years

18. Algebra club is growing exponentially at a rate of 5% each week. Algebra Club started its very first week ever with 6 students. How many will there be in 20 weeks?

19. What would be a reasonable domain and range for #18?

For each of the following:

- A. is this exponential growth or decay?
- B. identify the rate of growth or decay
- C. identify the y-intercept

22. $y = 4(0.3)^x$

A. _____

B. _____

C. _____

23. $Y = 0.5(1.25)^x$

A. _____

B. _____

C. _____

24. $y = 5(2)^x$

A. _____

B. _____

C. _____

25. $Y = 6(0.5)^x$

A. _____

B. _____

C. _____

26.

| | | | | | | | |
|---|--------|--------|------|---|-----|------|-------|
| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| y | .48828 | .78125 | 1.25 | 2 | 3.2 | 5.12 | 8.192 |

A. _____ B. _____ (hint $\frac{y_2}{y_1}$) C. _____

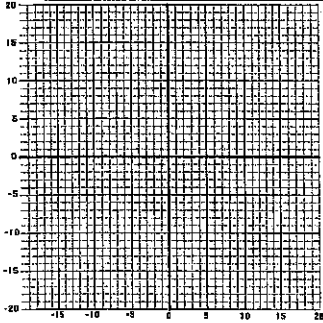
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| | | | | | | | |
|---|---------|-------|----|----|-----|------|-------|
| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| y | 281.125 | 112.5 | 45 | 18 | 7.2 | 2.88 | 1.152 |

A. _____ B. _____ C. _____

28. Make a table of values and graph $y = 8(0.7)^x$ and $y = 4(1.6)^x$

| | | | | | | |
|----------------|----|----|---|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| $y = 8(0.7)^x$ | | | | | | |
| $y = 4(1.6)^x$ | | | | | | |



| | $y = 8(0.7)^x$ | $y = 4(1.6)^x$ |
|--|----------------|----------------|
| Is this growth or decay? | | |
| What is the rate of growth or decay? (%) | | |
| What is the y-intercept? | | |
| What is the x-intercept? | | |
| What is the asymptote? | | |
| What is the domain? | | |
| What is the range? | | |

Where do the graphs intersect? _____

29. $f(x)$ is an exponential function, and $g(x)$ is a linear function. What is the smallest integer value of x where $f(x) > g(x)$

| | | | | |
|------|---|---|------|-------|
| x | 0 | 1 | 2 | 3 |
| f(x) | 6 | 9 | 13.5 | 20.25 |

| | | | | |
|------|----|----|----|----|
| x | 0 | 1 | 2 | 3 |
| g(x) | 12 | 17 | 22 | 27 |

$f(x)$ is an exponential function, and $g(x)$ is a linear function. What is the smallest integer value of x where $f(x) > g(x)$

| | | | | |
|------|---------------|---------------|----------------|----------------|
| x | 0 | 1 | 2 | 3 |
| f(x) | $\frac{3}{7}$ | $\frac{6}{7}$ | $1\frac{5}{7}$ | $3\frac{3}{7}$ |

| | | | | |
|------|---|---|---|---|
| x | 0 | 1 | 2 | 3 |
| g(x) | 1 | 3 | 5 | 7 |

A. Properties of Exponents

$a^m \cdot a^n = a^{m+n}$ if you multiply the bases, add the exponents

$(a^m)^n = a^{m \cdot n}$ if you raise a power to another exponent, multiply the exponents

$\frac{a^m}{a^n} = a^{m-n}$ if you divide the bases, subtract the exponents

$a^0 = 1$ $a^{-n} = \frac{1}{a^n}$ a negative exponent is the symbol to take the reciprocal

1. $\frac{18a^2b^3}{9a^2b^2} = 2b$ 2. $\frac{20ab^2c^2}{12a^2b^2c^2} = \frac{5b}{3a}$ 3. $(5x^2y^3z^2)^0 = 1$ 4. $5x^2(2x^2)^3 = 10x^{10}$

5. $(4x^2y^3z)^2 = 16x^4y^6z^2$ 6. $(2x^2y^3)^2(3xy^2)^3 = 4x^8y^6 \cdot 27x^3y^6 = 108x^{11}y^{12}$ 7. $(5x^2)^3(2x^2)^2 = 125x^6 \cdot 4x^4 = 500x^{10}$

8. Examine the solution #7. Identify the following:

Coefficient 500 base x exponent 34

Write each of the following with a single power:

9. $100x^2y^2 = (10xy)^2$ 10. $27x^3y^3 = (3xy)^3$ 11. $125a^3b^3 = (5ab)^3$ 12. $36x^2y^2 = (6xy)^2$

Solve for x: (hint - identify the base that is larger and rewrite it as a power.)

13. $2^{4x} = 32^2$
 $2^{4x} = (2^5)^2$
 $2^{4x} = 2^{10}$
 $4x = 10$
 $x = 2.5$

14. $3^{8x} = 81^{12}$
 $3^{8x} = (3^4)^{12}$
 $3^{8x} = 3^{48}$
 $8x = 48$
 $x = 6$

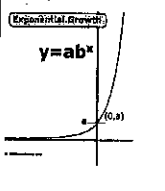
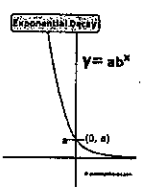
15. $2^x = 64^8$
 $2^x = (2^6)^8$
 $2^x = 2^{48}$
 $x = 48$

20. Eppler's centipede population is doubling every 5 months. Right now there are 3 centipedes. How many will there be in 60 months?

$a = 3$ $x = \frac{60}{5} = 12$ cycles $y = 3 \cdot 2^{12} = 12,288$

21. The half life of Byron-19 is 3 years. If you have 100 mg of Byron-19 right now:

- A. what is an equation that models this situation? $y = 100(.5)^{x/3}$
- B. How much Byron-19 will there be in 12 years? $y = 100(.5)^4 = 6.25$
- C. How much Byron-19 will there be in 36 years? $y = 100(.5)^{12} = .02$
- D. How much Byron-19 was there 6 years ago? $y = 100(.5)^{-2} = 400$

| Exponential growth | Exponential decay |
|---|---|
| $Y = ab^x$ | $Y = ab^x$ |
| $a =$ initial/starting amount | $a =$ initial/starting amount |
| $b =$ growth factor, $b > 1$ | $b =$ decay factor, $0 < b < 1$ |
|  |  |

16. Evaluate $y = 4^x$ for the domain $\{-2, -1, 0, 1, 2, 3\}$. No decimals!

$4^{-2} = \frac{1}{16}$ $4^{-1} = \frac{1}{4}$ $4^0 = 1$ $4^1 = 4$ $4^2 = 16$ $4^3 = 64$

17. Compound interest: you can use $y = ab^{x/n}$ OR $B = P(1+r)^x$

a and $P =$ starting amount

b or $r =$ interest rate per period. $\frac{\%}{\text{number of times per year compounded}}$

$x =$ number of times interest is compounded $(\text{number of years passed} \cdot \text{number of times per year compounded})$

- A. Invest \$500 at 3% annual for 12 years
 $a = 500$ $x = 12 \cdot 1 = 12$ $500(1.03)^{12} = \$712.88$
 $b = 3\% = .03$
- B. Invest \$500 at 3% semiannual for 12 years
 $a = 500$ $x = 12 \cdot 2 = 24$ $500(1.015)^{24} = \$714.75$
 $b = 3\% \div 2 = 1.5\%$
- C. Invest \$500 at 3% quarterly for 12 years
 $a = 500$ $x = 12 \cdot 4 = 48$ $500(1.0075)^{48} = \$715.70$
 $b = 3\% \div 4 = .75\%$
- D. Invest \$500 at 3% monthly for 12 years
 $a = 500$ $x = 12 \cdot 12 = 144$ $500(1.0025)^{144} = \$716.34$
 $b = 3\% \div 12 = .25\%$

18. Algebra Club is growing exponentially at a rate of 5% each week. Algebra Club started its very first week ever with 6 students. How many will there be in 20 weeks?

$a = 6$ $y = 6(1.05)^x$
 $b = 100\% + 5\% = 105\% = 1.05$ $y = 6(1.05)^{20} = 15.9$

19. What would be a reasonable domain and range for #18?

domain: weeks since Algebra Club did not exist prior to time 0 $x \geq 0$

range: students: since you can't have partial students, or negative students, all real numbers and integers are not valid. the least # students is 6, so whole numbers ≥ 6

For each of the following:

A. is this exponential growth or decay?

B. Identify the rate of growth or decay

C. identify the y-intercept

22. $y = 4(0.3)^x$

- A. decay
- B. $100\% - 30\% = 70\%$
- C. 4

23. $y = 0.5(1.25)^x$

- A. growth
- B. $125\% - 100\% = 25\%$
- C. 0.5

24. $y = 5(2)^x$

- A. growth
- B. doubling / 2
- C. 5

25. $y = 6(0.5)^x$

- A. decay
- B. half, 0.5
- C. 6

26.

| | | | | | | | |
|---|--------|--------|------|---|-----|------|-------|
| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| y | .48828 | .78125 | 1.25 | 2 | 3.2 | 5.12 | 8.192 |

- A. growth
- B. $\frac{b-100\%}{100\%} = \frac{1.25-100\%}{100\%} = 25\%$
- C. 2

27.

| | | | | | | | |
|---|---------|-------|----|----|-----|------|-------|
| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| y | 281.125 | 112.5 | 45 | 18 | 7.2 | 2.88 | 1.152 |

- A. decay
- B. $\frac{b-100\%}{100\%} = \frac{18-100\%}{100\%} = -40\%$
- C. 18

extend table or use graph calc.

29. $f(x)$ is an exponential function, and $g(x)$ is a linear function. What is the smallest integer value of x where $f(x) > g(x)$?

at $x=5$
 $f(x) = 45.6$
 $g(x) = 37$

| | | | | |
|------|---|---|------|-------|
| x | 0 | 1 | 2 | 3 |
| f(x) | 6 | 9 | 13.5 | 20.25 |

$$\frac{y_2}{y_1} = \frac{9}{6} = 1.5$$

$$\frac{13.5}{9} = 1.5$$

$$\frac{20.25}{13.5} = 1.5$$

$f(x) = 6(1.5)^x$

| | | | | |
|------|----|----|----|----|
| x | 0 | 1 | 2 | 3 |
| g(x) | 12 | 17 | 22 | 27 |

+5 +5 +5

$$y = 5x + 12$$

$f(x)$ is an exponential function, and $g(x)$ is a linear function. What is the smallest integer value of x where $f(x) > g(x)$?

| | | | | |
|------|---------------|---------------|----------------|----------------|
| x | 0 | 1 | 2 | 3 |
| f(x) | $\frac{3}{7}$ | $\frac{6}{7}$ | $\frac{12}{7}$ | $\frac{24}{7}$ |

$$\frac{\frac{6}{7}}{\frac{3}{7}} = 2 \quad \frac{\frac{12}{7}}{\frac{6}{7}} = 2$$

$$\frac{\frac{24}{7}}{\frac{12}{7}} = 2$$

$$y = \frac{3}{7}(2)^x$$

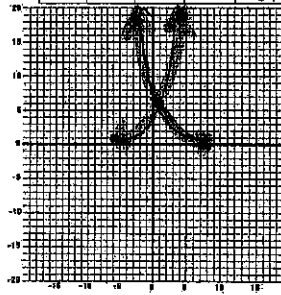
| | | | | |
|------|---|---|---|---|
| x | 0 | 1 | 2 | 3 |
| g(x) | 1 | 3 | 5 | 7 |

$$y = 2x + 1$$

at $x=5$
 $f(x) = 13.714$ or $13\frac{5}{7}$
 $g(x) = 11$

28. Make a table of values and graph $y = 8(0.7)^x$ and $y = 4(1.6)^x$

| | | | | | | |
|----------------|------|------|---|-----|------|------|
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| $y = 8(0.7)^x$ | 16.3 | 11.4 | 8 | 5.6 | 3.9 | 2.7 |
| $y = 4(1.6)^x$ | 1.5 | 2.5 | 4 | 6.4 | 10.2 | 16.4 |



| | $y = 8(0.7)^x$ | $y = 4(1.6)^x$ |
|--|----------------|----------------|
| Is this growth or decay? | decay | growth |
| What is the rate of growth or decay? (%) | 30% | 60% |
| What is the y-intercept? | 8 | 4 |
| What is the x-intercept? | none! | none! |
| What is the asymptote? | x-axis | x-axis |
| What is the domain? | \mathbb{R} | \mathbb{R} |
| What is the range? | $y > 0$ | $y > 0$ |

Where do the graphs intersect? $(2.8, 5.9)$
 (estimate!)

9 1 2 3 4

5 6 7 8

9 10 11 12

13 14 15 16

17 18 19 20